1. For $x > 0, n \in \mathbb{N}$, let $A = \{t \in \mathbb{R} : t > 0, t^n < x\}$. Let $y = \sup(A)$. Show that $y^n > x$ is not possible.

2. Decide which one of the following sequences are cauchy.
i) $\sum_{k=1}^{n} \frac{1}{k}$, (ii) $\frac{(-1)^n}{n}$, (iii) $\sum_{k=1}^{n} \frac{1}{k^2}$ and, (iv) $(-1)^n$

3. Decide (giving adequate justification via a proof or counter-example) whether the following statements are true or false:-

(a)Let A be a non-empty set of real numbers which is bounded below. Let

$$-A := \{x \in \mathbb{R} : -x \in A\}.$$

Then $\inf(A) = -\sup(-A)$.

(b) Let $\alpha > 1$. Let $f : [0, \infty) \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x^\alpha \sin(\frac{1}{x}) & x \neq 0, \\ 0 & x = 0. \end{cases}$$

Then f is differentiable at 0.

(c)Let $f, g : \mathbb{R} \to \mathbb{R}$. Let $E \subset \mathbb{R}$. If $f(x) \leq g(x)$ for all x in E, then

$$\inf_{x \in E} f(x) \geq \inf_{x \in E} g(x).$$

(d) All bounded sequences $\{z_n\}_{n=1}^{\infty}$ converge.

(e) Let $a \in \mathbb{R}$ be a limit point of a set $A \subset \mathbb{R}$. Let $\delta > 0$. Then $A \cap (a - \delta, a + \delta)$ is always finite.

4. Let $f : \mathbb{R} \to \mathbb{R}$ and suppose that

$$| f(x) - f(y) | \leq (\sin(x - y))^2$$

Show that f is differentiable and identify f.

5. Let $f : (-\infty, \infty) \to \mathbb{R}$ be a function given by $f(x) = e^x$. It is known that the f is differentiable at all $x \in \mathbb{R}$ and $f'(x) = f(x)$ for all $x \in \mathbb{R}$. Show that

$$e^{-x} < 1 - x + \frac{x^2}{2}$$

for all $x > 0$.